



CANDIDATE
NAME

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CENTRE
NUMBER

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NUMBER

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9231/23

May/June 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 (a)** Find the Maclaurin series for $\sin^{-1}x$ up to and including the term in x^3 . [5]

[illegible]

- (b)** Deduce an approximation to $\int_0^{\frac{1}{5}} \frac{1}{\sqrt{1-u^2}} du$, giving your answer as a fraction. [1]

[illegible]

2 The variables x and y are related by the differential equation

$$6\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x = t^2 + 10t + 13.$$

(a) Find the general solution for x in terms of t . [6]

This image shows a full page of a document template designed for handwriting practice. It consists of a series of evenly spaced, horizontal dashed lines extending across the entire width of the page. There are no margins, text, or other markings present.

(b) State an approximate solution for large positive values of t . [1]

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- 3** By considering the binomial expansions of $\left(z + \frac{1}{z}\right)^4$ and $\left(z - \frac{1}{z}\right)^4$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cot^4 \theta = \frac{\cos 4\theta + a \cos 2\theta + b}{\cos 4\theta - a \cos 2\theta + b},$$

where a and b are integers to be determined.

[7]

[illegible]

4 The curve C has equation

$$4y^3 + (x+y)^6 = 109.$$

(a) Show that, at the point $(-4, 3)$ on C , $\frac{dy}{dx} = \frac{1}{17}$. [3]

[illegible]

- (b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-4, 3)$.

[5]

[illegible]

- 5 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$2 \cosh^2 x = \cosh 2x + 1. \quad [3]$$

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- (b) Find the solution of the differential equation

$$\frac{dy}{dx} + 2y \tanh x = 1$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. [8]

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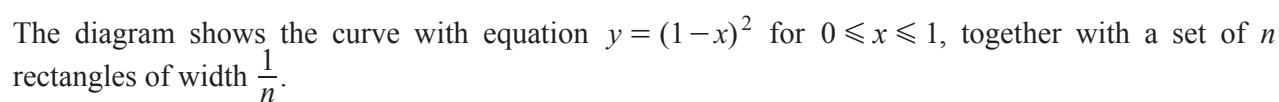
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- $$U_n = \frac{2n^2 + 3n + 1}{6n^2}. \quad [5]$$

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- (b)** Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (1-x)^2 dx$. [4]

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- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

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7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{4}{3}} (1+x^2)^{\frac{1}{2}n} dx$.

- (a) Find the exact value of I_{-1} giving your answer in the form $\ln a$, where a is an integer to be determined. [2]

[illegible]

- (b)** By considering $\frac{d}{dx} \left(x(1+x^2)^{\frac{1}{2}n} \right)$, or otherwise, show that

$$(n+1)I_n = nI_{n-2} + \frac{4}{3}\left(\frac{5}{3}\right)^n. \quad [5]$$

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) A curve has equation $y = x^2$, for $0 \leq x \leq \frac{2}{3}$. The arc length of the curve is denoted by s .

Use the substitution $u = 2x$ to show that $s = \frac{1}{2}I_1$ and find the exact value of s . [4]

[illegible]

8 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & -6a & 2a+2 \\ 0 & 1-a & 0 \\ 0 & 2-a & -1 \end{pmatrix}$$

where a is a constant with $a \neq 0$ and $a \neq 1$.

- (a) Show that the equation $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ has a unique solution and interpret this situation geometrically.

[3]

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- (b) Show that the eigenvalues of $\mathbf{A}$ are a , $1-a$ and -1 .

[2]

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- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^4 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [6]

[illegible]

- (d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^4$ in terms of $\mathbf{A}$ and a . [3]

This image shows a single sheet of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

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